Comparison of sampling methods and application to scatterometry

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Partial differential equations with uncertain input parameters are used in many applications in metrology, physics and engineering. The effect of input uncertainties on the solutions can be determined by the law of propagation of uncertainties. According to the guide to the expression of uncertainties in measurements (GUM) and its supplements, Monte Carlo sampling is recommended for nonlinear problems. In practice, large sampling sizes have to be chosen to ensure accuracy and precision. For computationally expensive problems only small sampling sizes are accessible. In this article we study and compare the propagation of uncertainties using three different sampling methods. The sampling methods chosen are Monte Carlo sampling, Latin hypercube sampling and a Sobol sequence based quasi Monte Carlo sampling. The methods are applied to the inverse problem of scatterometry with several simplifying assumptions in the measurement model. The solution of the inverse problem of scatterometry involves finite element solutions of a two dimensional Helmholtz equation. We found that among methods chosen Latin hypercube provides the most accurate and reliable results with respect to estimates of the geometry parameters, uncertainties and to repeatability.